E2.5 Signals & Linear Systems

Tutorial Sheet 9 – Discrete System Analysis

(Lectures 16 & 17)

1.* Using only the fact that $\gamma^k u[k] \Leftrightarrow \frac{z}{z-\gamma}$ and properties of the z-transform, find the z-transform of:

a) $k^2\gamma^k u[k]$

b) $n^3 u[n]$

c) $a^k \{u[k]-u[k-m]\}$

2.** By applying the time-shift property of z-transform, find the z-transform of the signal x[n] as shown in Fig. Q2. Hint: x[n] = n{u[n] – u[n-6]}.

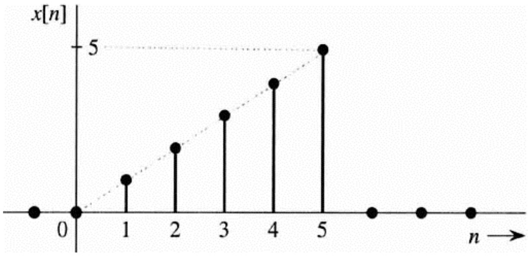


Fig Q2

3.* Show a canonical realization of the following transfer functions:

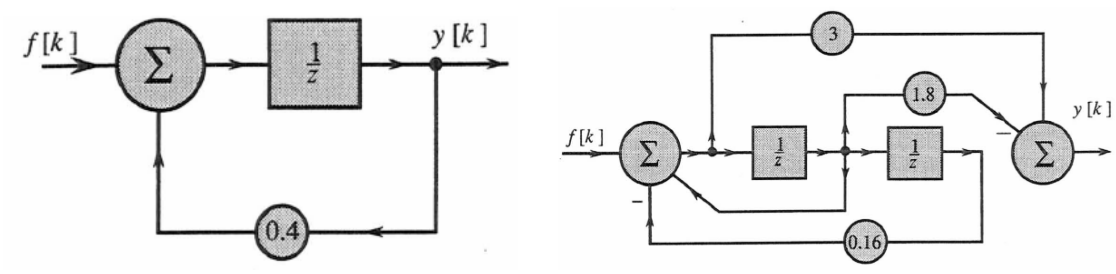
a) $H[z]=\frac{z(3z-1.8)}{z^2-z+0.16}$

b) $H[z]=\frac{3.8z-1.1}{(z-0.2)(z^2-0.6z+0.25)}$

4.** Draw a diagram showing the realization of a digital system whose transfer function is given by:

$$H[z]=\sum_{k=0}^{6} kz^{-k} \ .$$

5. * Derive the amplitude and phase response of the digital filters shown in Fig.Q5(a) and (b).



(a) (b)

Fig. Q5

6.** a) Realize a digital filter whose transfer function is give by

$$H[z] = K\frac{z+1}{z-a}.$$

b) Choose a value of K such that H[1] = 1. The amplitude response has a maximum value of $\Omega = 0$, and it decreases monotonically with frequency until $\Omega = \pi$. The 3-dB bandwidth is the frequency where the amplitude response drops to 0.707. Determine the 3-dB bandwidth of this filter when a = 0.2.

7.** Pole-zero configurations of two filters are shown in Fig. Q7(a) and (b). Sketch roughly the amplitude and the phase responses of these filters.

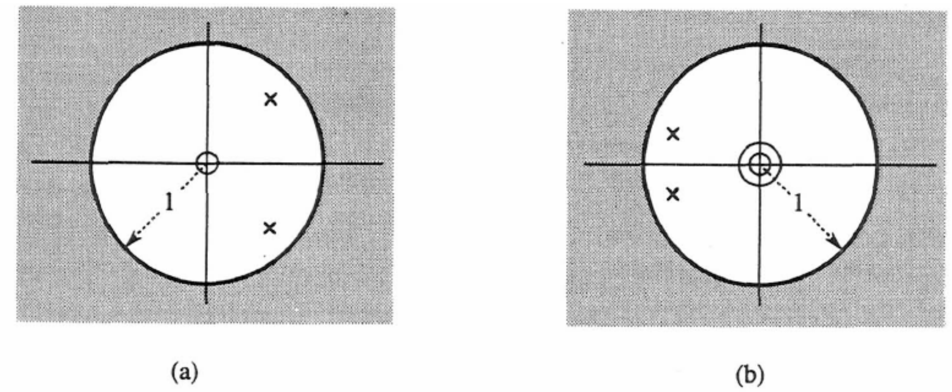


Fig. Q7

8.** Design a digital notch filter to reject frequency 5000 Hz completely, and to have a sharp recovery on either sides of 5000 Hz to a gain of unity. Assume that the sampling frequency is 40 kHz.

9.*** a) Show that the amplitude response of a system with a pole at z = r and a zero at z = 1/r (r is less than or equal to 1) is constant with frequency (this is called an "allpass" filter).

b) Generalize the result from a) to show that a digital LTI system with two poles at $z = re^{\pm j\theta}$ and two zeros at $z = \frac{1}{r}e^{\pm j\theta}$ $(r \leq 1)$ is also an allpass filter.